Tuning Sat4j PB Solvers for Decision Problems

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CRIL, Univ Artois & CNRS

Pseudo-Boolean (PB) Constraints

PB solvers generalize SAT solvers to take into account

- normalized PB constraints $\sum_{i=1}^{n} a_i l_i \geq d$
- cardinality constraints $\sum_{i=1}^{n} l_i \geq d$
- clauses $\sum_{i=1}^{n} l_i \geq 1 \equiv \bigvee_{i=1}^{n} l_i$

in which

- the coefficients a_i are non-negative integers
- each l_i is a literal, i.e., a variable v or its negation $\bar{v} = 1 - v$
- the degree d is a non-negative integer

Generalized Resolution

The generalized resolution proof system [Hooker, 1988] is used as the counterpart of the resolution proof system in PB solvers such as *Sat4j*

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d_1 \qquad b\bar{l} + \sum_{i=1}^{n} b_i l_i \geq d_2}{\sum_{i=1}^{n} (ba_i + ab_i) l_i \geq bd_1 + ad_2 - ab} \text{ (cancellation)}$$

$$\frac{\sum_{i=1}^{n} a_i l_i \geq d}{\sum_{i=1}^{n} \min(a_i, d) l_i \geq d} \text{ (saturation)}$$

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These two rules are used during conflict analysis to learn new constraints, but have very different properties compared to the resolution proof system used in classical SAT solvers

Preserving Conflicts

Analyzing Conflicts

Suppose that we have the following constraints:

$6\bar{b} + 6c + 4e + f + g + h \geq 7$ $\qquad$ $5a + 4b + c + d \geq 6$

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This conflict is analyzed by applying the cancellation rule as follows:

$$\frac{6\bar{b} + 6c + 4e + f + g + h \geq 7 \qquad 5a + 4b + c + d \geq 6}{15a + 15c + 8e + 3d + 2f + 2g + 2h \geq 20}$$

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The constraint we obtain here is no longer conflicting!

Weakening

To preserve the conflict, the weakening rule must be used:

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d}{\sum_{i=1}^{n} a_i l_i \geq d - a} \text{ (weakening)}$$

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Weakening can be applied in many different ways!

The Original Weakening Strategy

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This property gives an upper-bound of the slack of the produced constraint without actually computing the cancellation, its cost is not negligible as the operation must be repeated multiple times

An Important Property

In some cases, we do not need to estimate the slack, as we are sure that the constraint that will be derived will be conflicting

As soon as the coefficient of the literal to cancel is equal to 1 *in at least one of the constraints, the derived constraint is guaranteed to be conflicting [Dixon, 2004]*

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Different weakening strategies allow to do so

Disclaimer

The weakening strategies that follow are not applied at each derivation step during conflict analysis, but only when the coefficient of the pivot is not equal to 1 in both the conflict and in the reason, as otherwise we are sure that the conflict will be preserved by the previous property

Weakening Ineffective Literals

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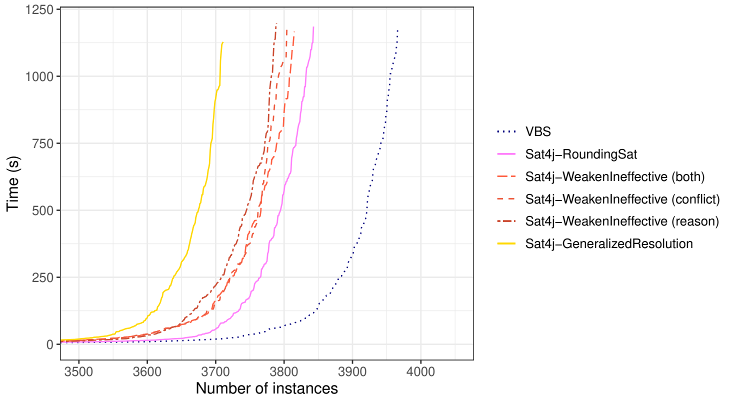
We propose here to apply it on one side of the cancellation, to infer stronger constraints and preserve PB reasoning

Weakening Ineffective Literals (Experiments)



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Once again, we propose here to apply this operation on only one side of the cancellation

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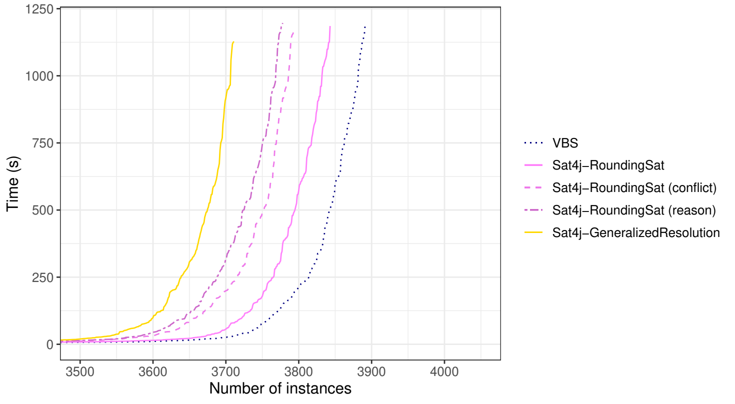

Using Partial Weakening

Another possibility is to consider a variant of the weakening rule, known as partial weakening.

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d \qquad k \in \mathbb{N} \qquad 0 < k \leq a}{(a-k)l + \sum_{i=1}^{n} a_i l_i \geq d-k} \text{ (partial weakening)}$$

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In general, this rule allows to derive stronger constraints than with the weakening rule.

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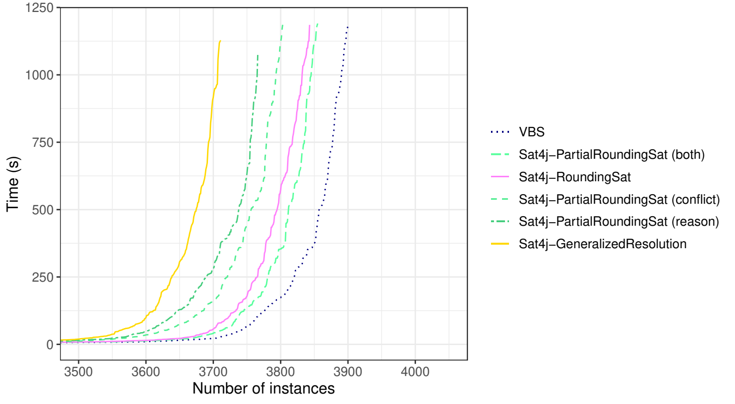

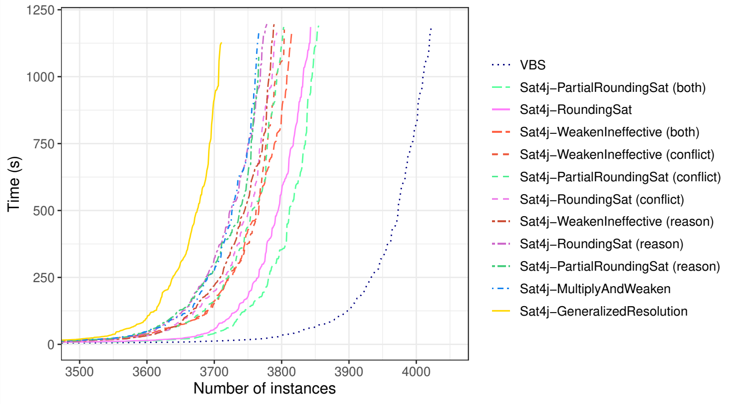

1250
1000
750
500
250
0
Time (s)
3500
3600
3700
3800
3900
4000
Number of instances
VBS
Sat4j–PartialRoundingSat (both)
Sat4j–RoundingSat
Sat4j–WeakenIneffective (both)
Sat4j–WeakenIneffective (conflict)
Sat4j–PartialRoundingSat (conflict)
Sat4j–RoundingSat (conflict)
Sat4j–WeakenIneffective (reason)
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Sat4j–MultiplyAndWeaken
Sat4j–GeneralizedResolution

Choosing Decision Variables

A Conflict Analysis

Suppose that we have the following constraints:

$$3\bar{a}(?@?) + 3\bar{f}(?@?) + d(?@?) + e(?@?) \geq 5$$

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We now apply the cancellation rule between these two constraints:

$$\frac{3\bar{a} + 3\bar{f} + d + e \geq 5 \qquad 6a + 3b + 3c + 3d + 3f \geq 9}{3a(0@3) + 3b(1@1) + 3c(0@2) + 2\bar{d}(1@3) + e(?@?) \geq 7}$$

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The PB constraints involved in this conflict analysis have very different properties compared to clauses!

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All variables encountered during conflict analysis are bumped

This is the case for all the variables appearing in the previous reason:

$$3\bar{a} + 3\bar{f} + d + e \geq 5$$

This means that the scores of the variables a, f, d and e are incremented

(E)VSIDS for Making Decisions: Coefficients (1)

A first approach for adapting VSIDS to PB constraints has been proposed in [Dixon, 2004], but it only takes into account the original cardinality constraints, by incrementing the score of each variable by the value of the degree

$$a + b + c \geq 2$$

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However, this approach does not take into account the coefficients in a PB constraint, contrary to the implementation proposed in *Pueblo* [Sheini and Sakallah, 2006], which increments the score of the variables by the value of the coefficient of a variable divided by the degree (e.g., $3/5$ for a in the reason below)

$$3\bar{a} + 3\bar{f} + d + e \geq 5$$

(E)VSIDS for Making Decisions: Coefficients (2)

Considering again the constraint we used as a reason before

$$3\bar{a} + 3\bar{f} + d + e \geq 5$$

We propose to take its coefficients into account with 3 other strategies:

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- `bump-degree`: the score of each variable is incremented by the degree of the constraint (5 for all variables)

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We can also take into account the current assignement when bumping variables

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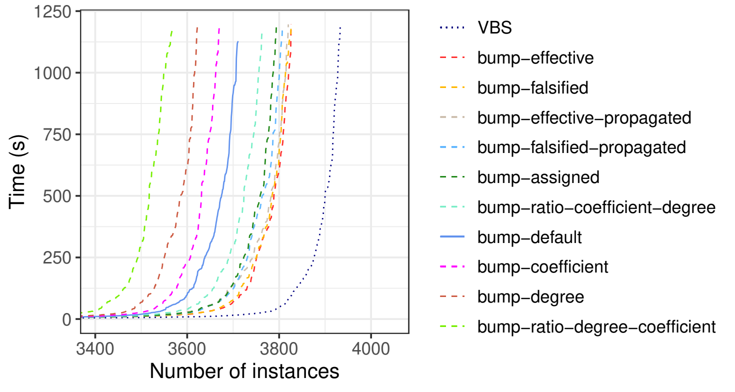
(E)VSIDS: Experiments (Sat4j-GR)



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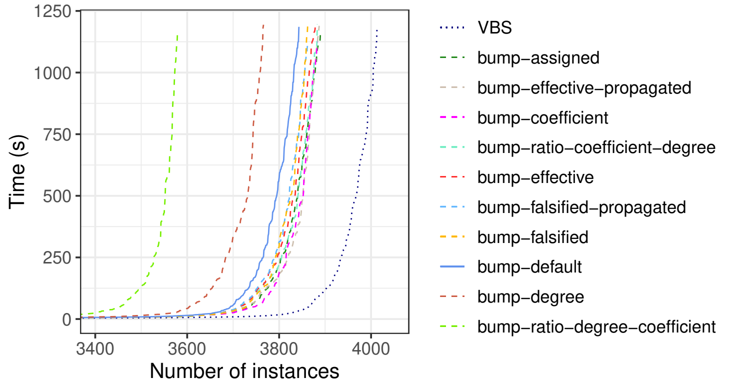
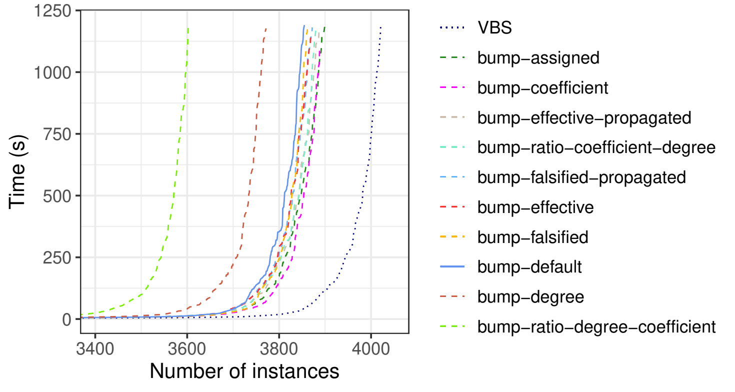


(E)VSIDS: Experiments (Sat4j-PartialRS)



Learned Constraint Quality

Quality of Learned Constraints: Classical Implementations

In SAT solvers, evaluating the quality of learned constraints is used to choose which constraints should be deleted and to decide when a restart should be triggered

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The quality measures used by SAT solvers do not take into account the properties of PB constraints

Quality of Learned Constraint: Size and Coefficients (1)

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Quality of Learned Constraint: Size and Coefficients (2)

Another indicator that we have for evaluating the quality of a constraint is to estimate its strength with its slack

$$3a + 3b + 3c + 2\bar{d} + e \geq 7$$

In this case, we prefer to consider the *absolute* slack of the constraint, independantly of the current assignment: in this example, it is equal to 5 (while, under the current assignment, it is equal to -1)

We consider quality measures based on the value of the slack of the constraints: the lower the slack, the better the constraint

Quality of Learned Constraint: Assignments (LBD)

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There are satisfied and unassigned literals in this constraint!

We thus introduce 4 new definitions of LBD:

- `lbd-a`: the LBD is computed over assigned literals only
- `lbd-s`: the LBD is computed over assigned literals, and unassigned literals are considered assigned at the same (dummy) decision level
- `lbd-d`: the LBD is computed over assigned literals, and unassigned literals are considered assigned at different (dummy) decision levels
- `lbd-f`: the LBD is computed over falsified literals only
- `lbd-e`: the LBD is computed over effective literals only

Quality of Learned Constraint: Deletion

Deleting constraints is required by SAT solvers to limit the memory usage and to prevent unit propagation from slowing down

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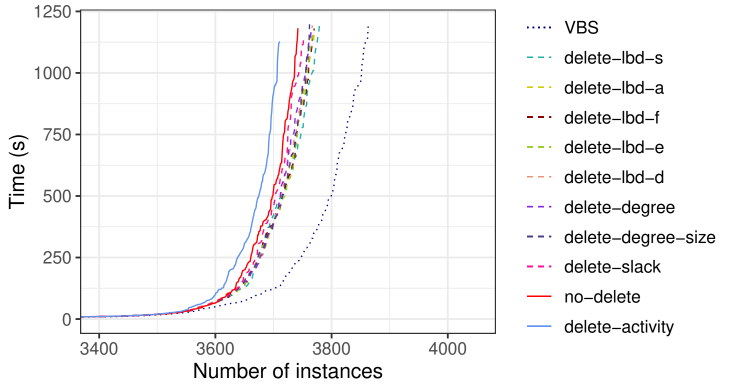
The constraints to delete are those having a bad score w.r.t. the quality measure used in the solver

We thus introduce the following deletion strategies:

- `delete-degree`
- `delete-degree-size`
- `delete-slack`
- `delete-lbd-a`
- `delete-lbd-s`
- `delete-lbd-d`
- `delete-lbd-f`
- `delete-lbd-e`

Learned Constraint Deletion: Experiments (Sat4j-GR)



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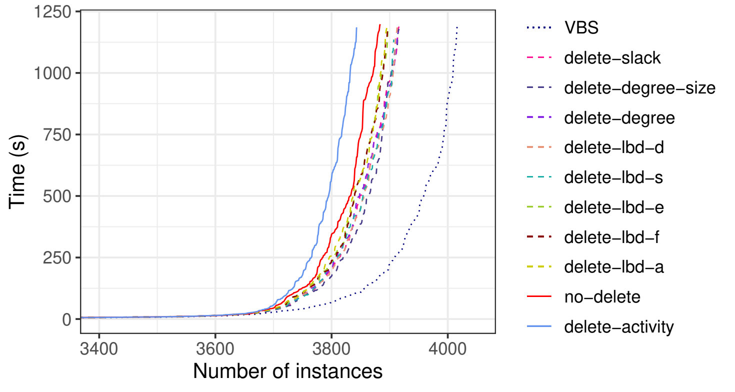
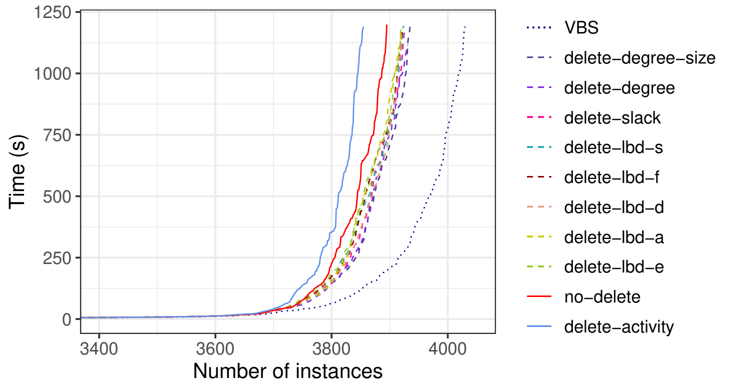


Learned Constraint Deletion: Experiments (Sat4j-PartialRS)



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Whenever the most recent constraints are of poor quality compared to all the others, a restart is performed

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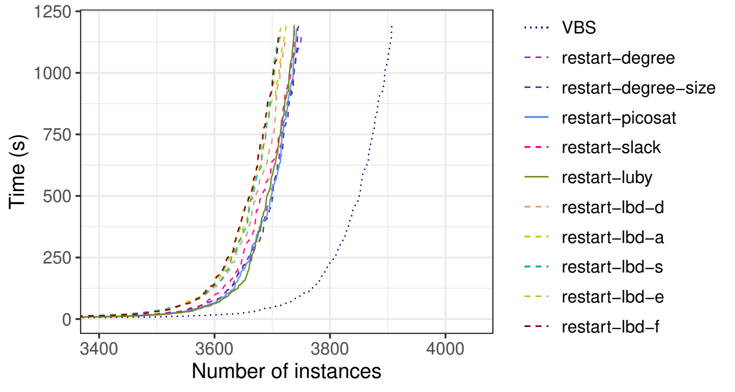
Following *Glucose*'s approach, we consider adaptive restarts based on the quality of recently learned constraints

> *Whenever the most recent constraints are of poor quality compared to all the others, a restart is performed*

We thus introduce the following restart strategies:

- `restart-degree`
- `restart-degree-size`
- `restart-slack`
- `restart-lbd-a`
- `restart-lbd-s`
- `restart-lbd-d`
- `restart-lbd-f`
- `restart-lbd-e`

Restarts: Experiments (Sat4j-GR)



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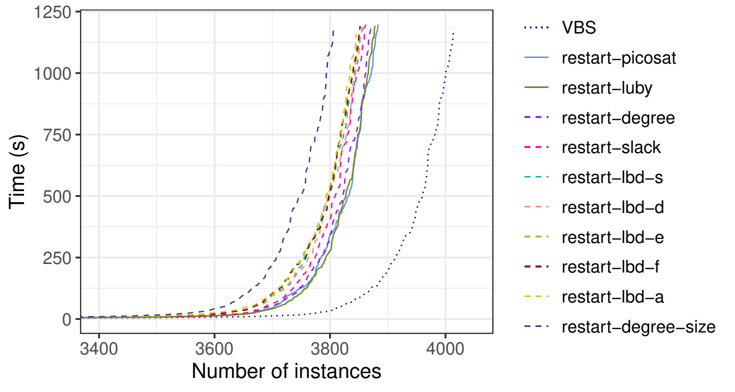

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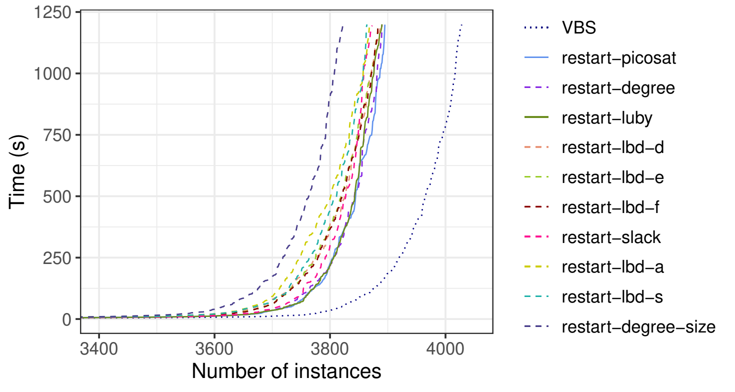


Combining the Best Strategies

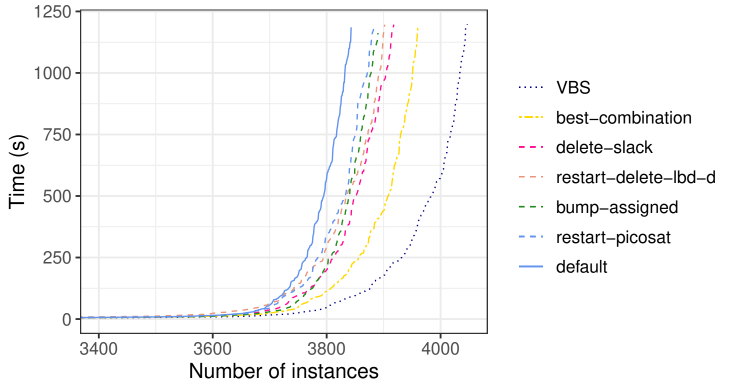
Combining the Best Strategies: Sat4j-GR

In Sat4j-GeneralizedResolution, the best strategies are

- `bump-falsified`
- `delete-lbd-s`
- `restart-degree`

Let us combine all these strategies!

Combining the Best Strategies: Sat4j-GR (Experiments)



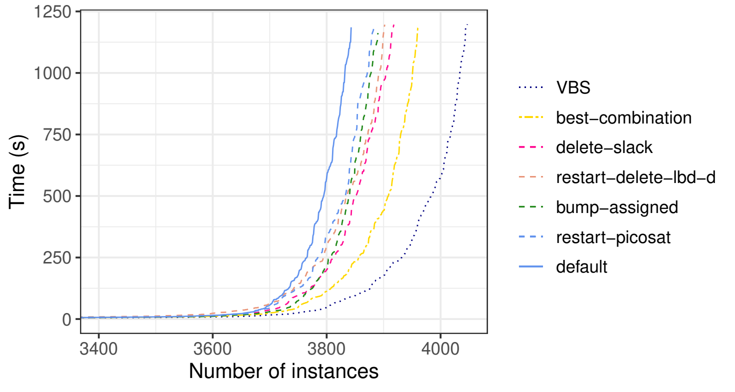
Combining the Best Strategies: Sat4j-RS

In Sat4j-RoundingSat, the best strategies are

- `bump-assigned`
- `delete-slack`
- `restart-picosat`

Let us combine all these strategies!

Combining the Best Strategies: Sat4j-RS (Experiments)



Combining the Best Strategies: Sat4j-PartialRS

In Sat4j-PartialRoundingSat, the best strategies are

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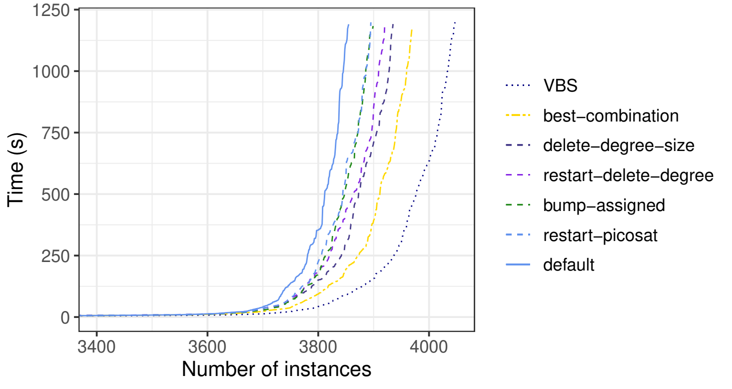
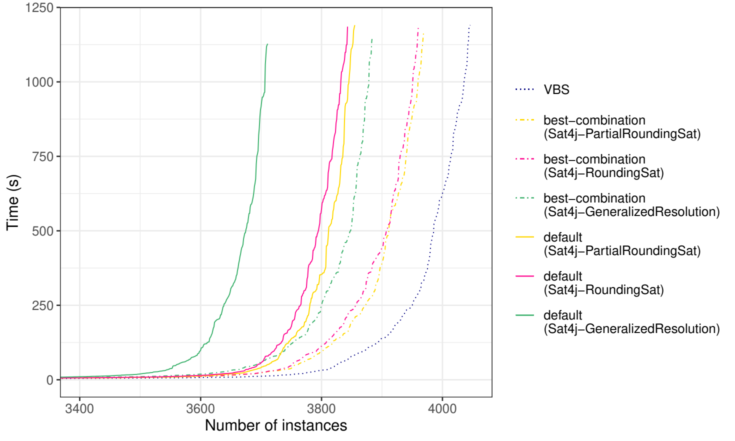
Let us combine all these strategies!

Combining the Best Strategies: Sat4j-PartialRS (Experiments)



Combining the Best Strategies: Complete Overview

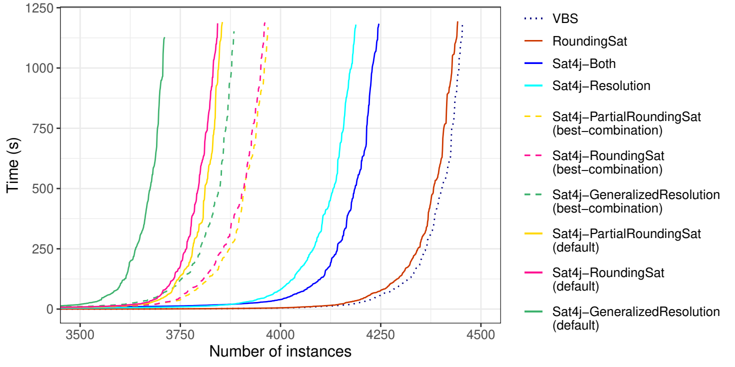


Conclusion and Perspectives

Conclusion

- CDCL in PB solvers requires a particular attention to preserve its properties compared to SAT solvers
- Different weakening strategies may be applied to preserve conflicts
- Bumping variables works better when considering the current assignment
- Considering the coefficients to evaluate the quality of a learned PB constraint provides a quite accurate measure

Disclaimer



Time (s)
1250
1000
750
500
250
0
3500
3750
4000
4250
4500
Number of instances
VBS
RoundingSat
Sat4j–Both
Sat4j–Resolution
Sat4j–PartialRoundingSat (best–combination)
Sat4j–RoundingSat (best–combination)
Sat4j–GeneralizedResolution (best–combination)
Sat4j–PartialRoundingSat (default)
Sat4j–RoundingSat (default)
Sat4j–GeneralizedResolution (default)

Perspectives

- Consider more specifically the impact of the weakening rule on either the conflict or the reason side of the cancellation rule
- Find better tradeoffs to combine the different weakening strategies
- Find better extension or combinations of the presented CDCL strategies
- Consider all the presented strategies on optimization problems

Tuning Sat4j PB Solvers for Decision Problems

Romain Wallon

Zoom Seminar – August 28th, 2020

CRIL, Univ Artois & CNRS