Pseudo-Boolean Reasoning and Compilation

PhD Defense

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UNIVERSITÉ D'ARTOIS

Boolean Satisfiability

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Modern SAT solvers can now deal with problems containing millions of variables and clauses

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On such instances, pseudo-Boolean reasoning and cutting planes based inference can offer better performance

Pseudo-Boolean Reasoning

Pseudo-Boolean (PB) Constraints

PB solvers are generalizations of SAT solvers that allow to consider

- normalized PB constraints $\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta$
- cardinality constraints $\sum_{i=1}^{n} \ell_i \geq \delta$
- clauses $\sum_{i=1}^{n} \ell_i \geq 1$

in which

- the coefficients α_i are non-negative integers
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We use the cutting-planes proof system to reason with such constraints

Cutting Planes and Generalized Resolution

PB solvers often use a subset of cutting planes rules known as Generalized Resolution [Hooker, 1988], which uses the following rules

$$\frac{\alpha\ell + \sum_{i=1}^{n} \alpha_i \ell_i \geq \delta_1 \qquad \beta\overline{\ell} + \sum_{i=1}^{n} \beta_i \ell_i \geq \delta_2}{\sum_{i=1}^{n} (\beta\alpha_i + \alpha\beta_i)\ell_i \geq \beta\delta_1 + \alpha\delta_2 - \alpha\beta} \text{ (cancellation)}$$

$$\frac{\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta}{\sum_{i=1}^{n} \min(\alpha_i, \delta)\ell_i \geq \delta} \text{ (saturation)}$$

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As with the resolution rule in classical SAT solvers, these two rules can be used to learn new constraints during conflict analysis

The Division Rule

Another useful rule is that of the division

$$\frac{\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta \qquad \rho \in \mathbb{N}^*}{\sum_{i=1}^{n} \lceil \frac{\alpha_i}{\rho} \rceil \ell_i \geq \lceil \frac{\delta}{\rho} \rceil} \text{ (division)}$$

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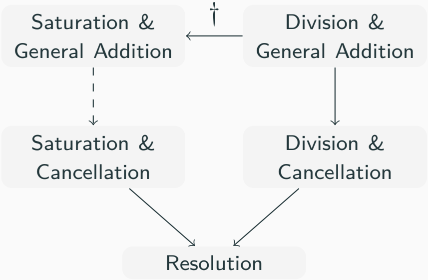
This rule may allow to strengthen a constraint over the reals and can be used to replace the saturation rule

Proof System Strength (on PB Inputs) [Vinyals et al., 2018]



$A \longrightarrow B$ if A is strictly stronger than B († on polynomial size coefficients)
$A \dashrightarrow B$ if any proof of B can be translated in polynomial time into a proof of A

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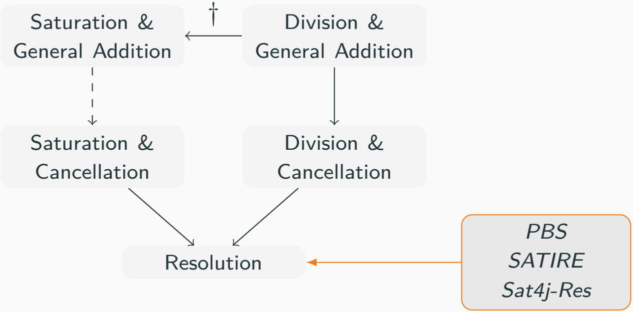

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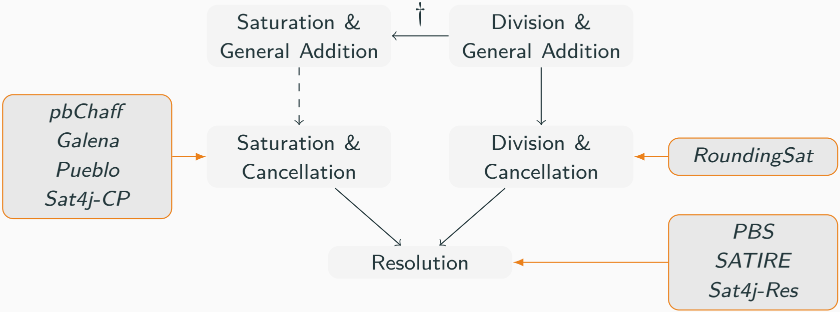

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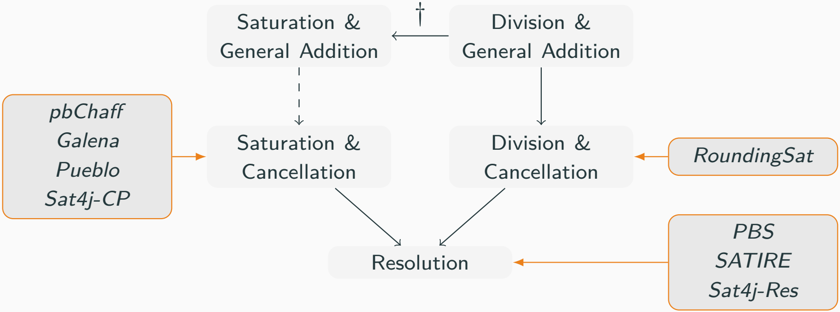

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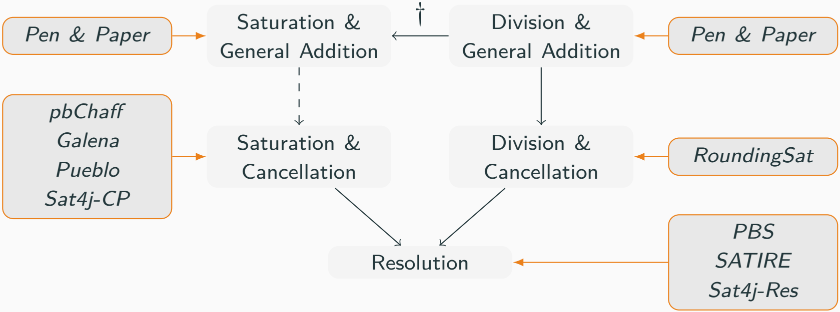

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A Gap Between Theory and Practice

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The rules of the proof system are applied based on the structure and semantics of the constraints

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The application of the cancellation rule in PB solvers is guided by the propagations that lead to a conflict

Properties of PB Constraints

Motivation

PB formulae can be grouped into different languages, depending on the kind of constraints they contain:

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Let us study the pros and cons of using PB constraints from a knowledge representation perspective

Queries [IJCAI'18]

	CO	VA	CE	IM	EQ	SE	CT	ME
CNF	∘	✓	∘	✓	∘	∘	∘	∘
CARD	∘	✓	∘	✓	∘	∘	∘	∘
PBC	∘	✓	∘	✓	∘	∘	∘	∘
NNF	∘	∘	∘	∘	∘	∘	∘	∘

✓ polynomial-time ∘ NP-hard

CO (COnsistency) Is a formula consistent?
VA (VAlidity) Is a formula valid?
CE (Clausal Entailment) Is a given clause implied by a formula?
IM (IMplication) Is a formula implied by a given cube/term?
EQ (EQuivalence) Are two formulas equivalent?
SE (Sentential Entailment) Is a formula entailed by an other one?
CT (CounTing) How many models does a formula have?

Transformations [IJCAI'18] (with more recent results)

	CD	FO	SFO	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
CNF	✓	▪	✓	✓	✓	▪	✓	▪
CARD	✓	▪	▪	✓	✓	▪	▪	▪
PBC	✓	▪	▪	✓	✓	▪	▪	▪
NNF	✓	○	✓	✓	✓	✓	✓	✓

✓ polynomial-time ○ NP-hard ▪ exponential-size

CD (ConDitioning) Compute $\phi|\tau$ where τ is a consistent cube/term
SFO (Singleton FOrgetting) Compute $\exists x\phi \equiv (\phi|x) \vee (\phi|\overline{x})$
FO (FOrgetting) Compute $\exists X\phi$ where X is a set of variables
$\wedge$**C (Closure under** $\wedge$**)** Compute $\bigwedge_{i=1}^{n} \phi_i$
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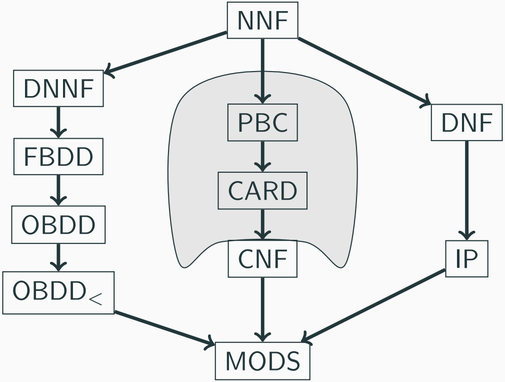
Succinctness [IJCAI'18]

Succinctness captures the ability of a language to represent information using little space



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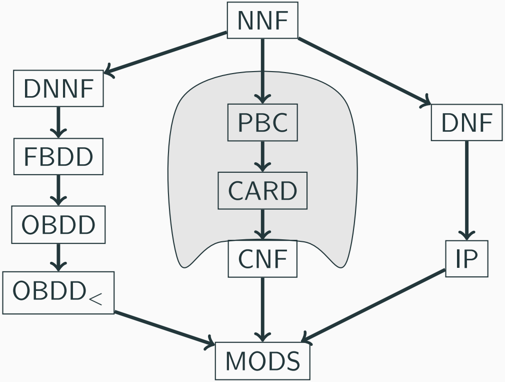

The main advantage of PB constraints is their succinctness w.r.t. clauses, and the reasoning power brought by the cutting planes proof system

An Achilles Heel in the Cutting Planes Proof System

Irrelevant Literals [IJCAI'20]

Cutting planes rules may introduce irrelevant literals

$$\frac{3d + a + b + c \geq 3 \qquad 3\bar{d} + 2a + 2b \geq 3}{3a + 3b + c \geq 3}$$

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A literal is said to be irrelevant in a PB constraint when its truth value does not impact the truth value of the constraint: irrelevant literals can thus be removed

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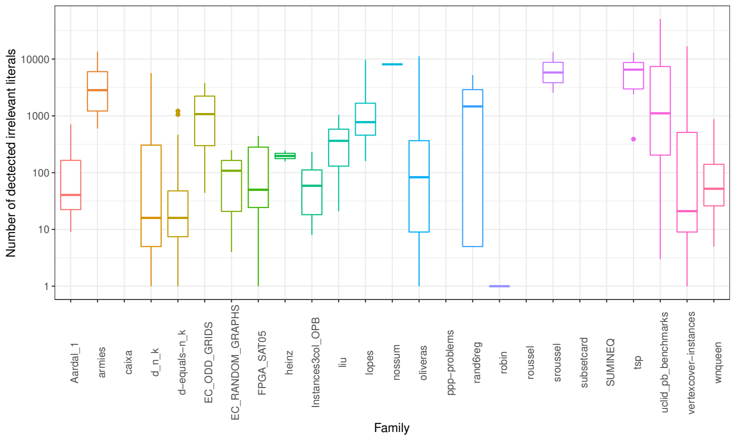
Production of Irrelevant Literals



Statistics about the production of irrelevant literals in *Sat4j-GeneralizedResolution* for each family of benchmarks (logarithmic scale)

Artificially Relevant Literals [IJCAI'20]

Irrelevant literals may become artificially relevant, in which case they may impact the strength of the derived constraints

$$\dfrac{\dfrac{3a + 3b + c \geq 3 \qquad 3\bar{a} + 3d + 2c \geq 3}{3b + 3c + 3d \geq 3}}{b + c + d \geq 1}$$

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Detecting irrelevant literals is NP-hard, we thus introduce an incomplete algorithm for removing them

Detecting Irrelevant Literals [IJCAI'20] (1)

Irrelevant literals can be detected thanks to this reduction to subset-sum

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A dynamic programming algorithm can decide whether any of the equalities has a solution in pseudo-polynomial time with a single run

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Our algorithm solves subset-sum modulo a fixed number, or even several numbers

Removing Irrelevant Literals [IJCAI'20]

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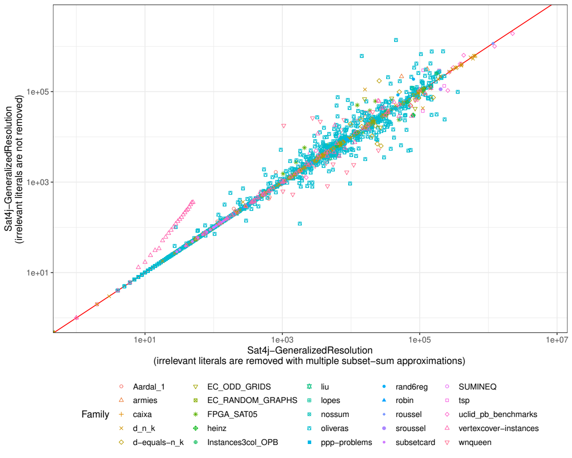
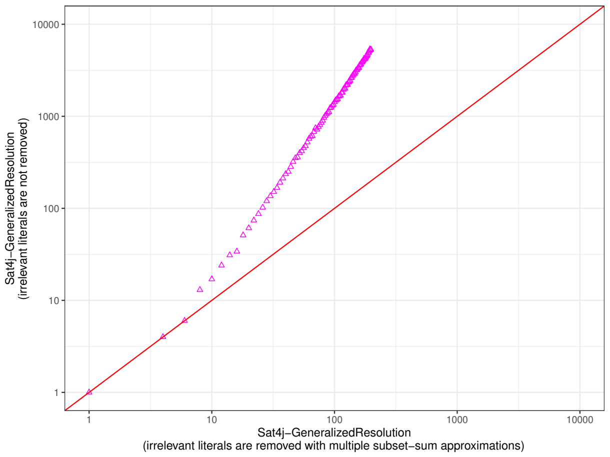
In practice, we use a heuristic based on the slack to decide which strategy to apply, as none of them is better than the other

Impact of the Removal of Irrelevant Literals on the Proof



Comparison of the size of the proofs (number of cancellations) built by *Sat4j-GeneralizedResolution* with and without the removal of irrelevant literals on all benchmarks (logarithmic scale)

Focus on the Vertex-Cover Family: Experimental Results



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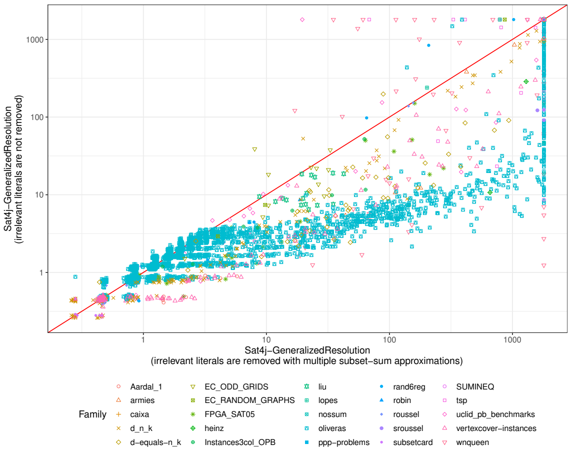
Even few irrelevant literals can lead to the production of an exponentially larger proof

Impact of the Removal of Irrelevant Literals on the Runtime



Comparison of the runtime of *Sat4j-GeneralizedResolution* with and without the removal of irrelevant literals on all benchmarks (logarithmic scale)

Leveraging Weakening

The weakening rules are defined as follows:

$$\frac{\alpha\ell + \sum_{i=1}^{n} \alpha_i \ell_i \geq \delta}{\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta - \alpha} \text{ (weakening)}$$

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These rules are already used by PB solvers to maintain invariants during conflict analysis

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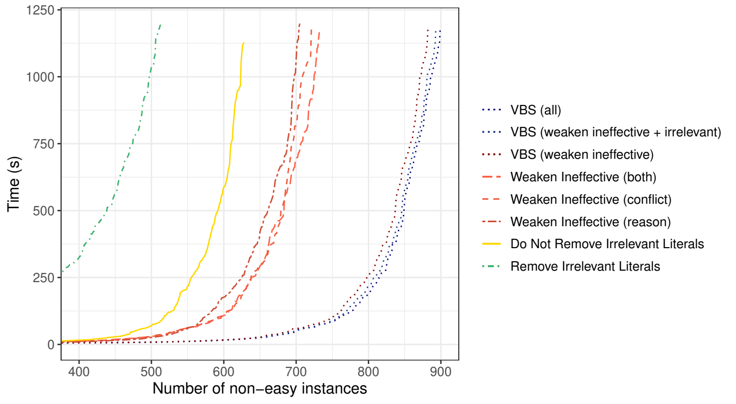
In the context of the current partial assignment, it is easy to detect ineffective literals, but they can only be weakened away (as ineffective literals may be relevant)

Experimental Results



Cactus plot of the different removal strategies of irrelevant literals

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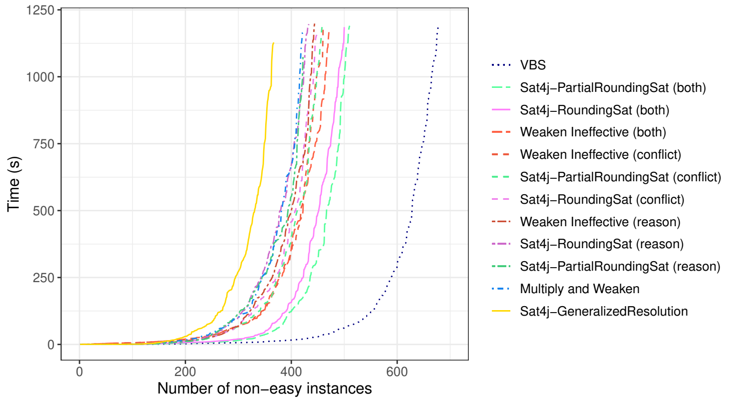
This operation may be applied on either one or both sides of the cancellation

Experiments



Comparison of the runtime of different weakening strategies

Fine Tuning of PB Solvers

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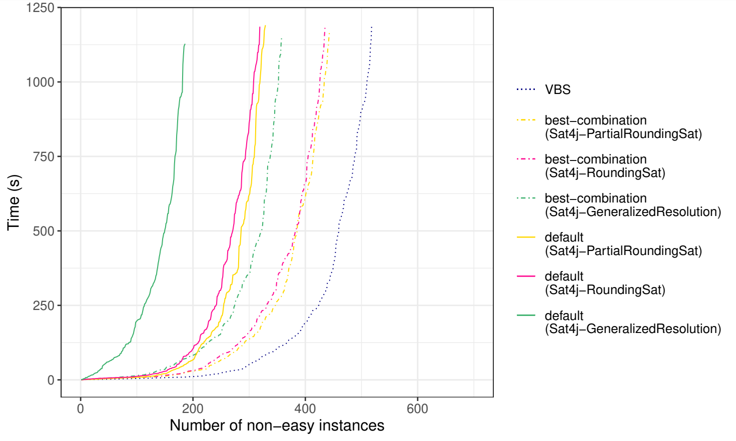
Our main finding is that considering the size of the coefficients and the current partial assignment allows to significantly improve the solver

Runtime of Sat4j with Different Configurations



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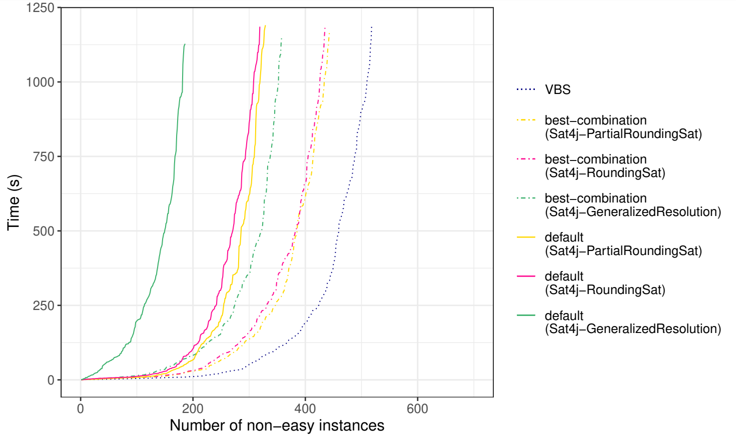
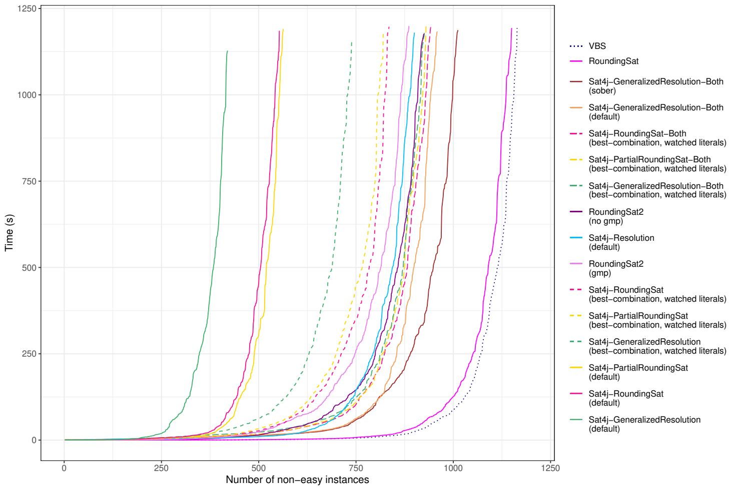


All configurations are improved by the combination of the new strategies

Comparison of Sat4j with RoundingSat



Conclusion and Perspectives

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- Complementary heuristics implemented in CDCL PB solvers can be adapted to take into account properties of PB constraints and to improve the performance of *Sat4j*

Perspectives

- Find other strategies for applying cutting planes rules so as to exploit more power of this proof system
- Design such strategies so as to prevent the production of irrelevant literals instead of removing them
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- Improve the detection of conflicts to deal with the conflictual reasons encountered during conflict analysis

Scientific Production: International Papers

D. Le Berre, P. Marquis, S. Mengel and R. Wallon. *Pseudo-Boolean Constraints from a Knowledge Representation Perspective.* Published at IJCAI'18.

S. Mengel and R. Wallon. *Revisiting Graph Width Measures for CNF-Encodings.* Published at SAT'19.

S. Mengel and R. Wallon. *Graph Width Measures for CNF-Encodings with Auxiliary Variables.* Published in JAIR (vol. 67, 2020).

D. Le Berre, P. Marquis and R. Wallon. *On Weakening Strategies for PB Solvers.* Published at SAT'20.

D. Le Berre, P. Marquis, S. Mengel and R. Wallon. *On Irrelevant Literals in Pseudo-Boolean Constraint Learning.* Published at IJCAI'20.

Scientific Production: Software

I am a committer of *Sat4j*[1] in which I implemented several features:

- Detection and removal of irrelevant literals
- Different weakening strategies (including *RoundingSat*'s)
- New heuristics dedicated to the resolution of PB problems

All these implementations have also been rigorously experimented and evaluated, before being presented in different venues

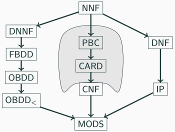
I also contributed to the development of *Metrics*[2], a Python library and app for analyzing experimental results

[1] `https://gitlab.ow2.org/sat4j/sat4j`
[2] `https://github.com/crillab/metrics`

Thanks for your attention! Questions?

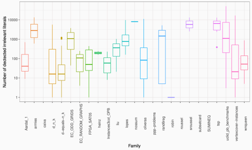
Succinctness [IJCAI'18]

Succinctness captures the ability of a language to represent information using little space



The main advantage of PB constraints is their succinctness w.r.t. clauses, and the reasoning power brought by the cutting planes proof system

Production of Irrelevant Literals



Statistics about the production of irrelevant literals in *Sat4j-GeneralizedResolution* for each family of benchmarks (logarithmic scale)

Weakening Ineffective Literals

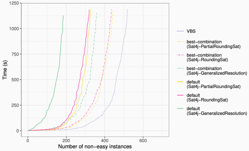
During conflict analysis, some literals may not play a role in the conflict being analyzed: it is thus possible to weaken them away while preserving invariants

$$\frac{\dfrac{\boxed{3\bar{a}} + 3\bar{b} + c + \boxed{d} + \boxed{e} \geq 6}{3\bar{b} + c \geq 6 - 3 - 1 - 1 = 1}}{\bar{b} + c \geq 1} \qquad \frac{\dfrac{2a + b + \boxed{c} + f \geq 2}{2a + b + f \geq 2 - 1 = 1}}{a + b + f \geq 1}$$

Ineffective literals can be seen as locally irrelevant, as opposed to the (globally) irrelevant literals presented before

In the context of the current partial assignment, it is easy to detect ineffective literals, but they can only be weakened away (as ineffective literals may be relevant)

Runtime of Sat4j with Different Configurations



All configurations are improved by the combination of the new strategies

Pseudo-Boolean Reasoning and Compilation

PhD Defense

Romain Wallon

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